NONSTATIONARY PLANE-PARALLEL FILTRATION IN AN INHOMOGENEOUS ZONAL SEAM

B. A. Suleimanov,[a] É. M. Abbasov,[a] and A. O. Éfendieva[b]

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A nonstationary plane-parallel filtration of liquid in an inhomogeneous zonal seam of finite extent has been investigated. A closed elastic liquid flow was considered. It has been established that the rate of this flow depends on its direction.

In [1, 2], a nonstationary filtration of a plane-radial liquid flow in an inhomogeneous zonal porous medium has been investigated. However, in these works, seams of infinitely large extent were considered or the problem was solved by approximate methods.

In the present work, a seam of finite extent was considered and the problem was solved by analytical methods. We investigated the influence of a jump-like change in the permeability of a porous seam of finite extent, arising as a result of a change in the direction of a liquid flow in it [3], on its filtration ability and on the rate of this flow. A closed elastic liquid flow was considered.

The initial equation has the form [1]

$$\frac{\partial^2 \Delta P_1}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta P_1}{\partial r} = \frac{1}{\chi_1}\frac{\partial \Delta P_1}{\partial t}\,, \quad \frac{\partial^2 \Delta P_2}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta P_2}{\partial r} = \frac{1}{\chi_2}\frac{\partial \Delta P_2}{\partial t}\,, \tag{1}$$

where $\Delta P_1 = P_s - P_1$ and $\Delta P_2 = P_s - P_2$.

The initial and boundary conditions (Fig. 1) are as follows:

$$\Delta P_1 = 0\,, \quad \Delta P_2 = 0 \quad \text{at} \quad t = 0\,; \tag{2}$$

$$k_1 \frac{\partial \Delta P_1}{\partial r} = k_2 \frac{\partial \Delta P_2}{\partial r} \quad \text{at} \quad r = R_1\,, \quad \Delta P_1 \big|_{r=R_1} = \Delta P_2 \big|_{r=R_1}\,, \quad \Delta P_1 \big|_{r=R_2} = 0\,, \quad \Delta P_1 \big|_{r=R_w} = P_s - P_f\,. \tag{3}$$

Integrating (1), we obtain

$$\Delta P_1 (r, t) = D_2 + D_1 \ln r + \sum_{\nu=1}^{\infty} \left[A_\nu J_0 \left(x_\nu \frac{r}{R_1} \right) + B_\nu Y_0 \left(x_\nu \frac{r}{R_1} \right) \right] \exp \left(-x_\nu^2 \frac{\chi_1 t}{R_1^2} \right), \tag{4}$$

$$\Delta P_2 (r, t) = D_2 + D_1 \ln r + \sum_{\nu=1}^{\infty} \left[A_\nu J_0 \left(x_\nu \frac{r}{R_w} \right) + B_\nu Y_0 \left(x_\nu \frac{r}{R_w} \right) \right] \exp \left(-x_\nu^2 \frac{\chi_2 t}{R_w^2} \right). \tag{5}$$

Solving Eqs. (4) and (5) with allowance for the boundary conditions (3), we find the transcendental equation [4–6]

[a]Gipromorneftegaz Scientific-Research Institute, 88 Zardabi Ave., Baku, Az1012, Azerbaidzan; [b]Institute of Mathematics and Mechanics, National Academy of Sciences of Azerbaidzan, Baku, Azerbaidzan. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 6, pp. 89–92, November–December, 2005. Original article submitted May 20, 2004; revision submitted October 12, 2004.

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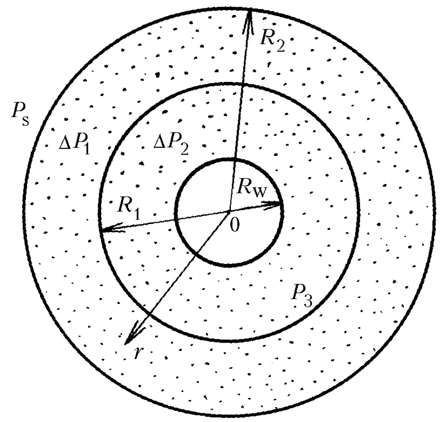

Fig. 1. Horizontal section of a seam.

$$J_0(x_\nu)\, Y_0\left(x_\nu \frac{R_1}{R_w}\right) - J_0\left(x_\nu \frac{R_1}{R_w}\right) Y_0(x_\nu) = 0\,, \tag{6}$$

from which the roots x_ν are determined. Then Eq. (5) takes the form

$$P_2(r, t) = P_s - \frac{\Delta P_{st}}{\ln \dfrac{R_w}{R_2}} \ln \frac{r}{R_2} - \pi \left(\ln \frac{R_2}{R_w}\right) \frac{\Delta P_{st}}{\ln \dfrac{R_w}{R_2}} \sum_{\nu=1}^{\infty} \frac{J_0\left(x_\nu \dfrac{R_2}{R_w}\right) J_0(x_\nu)}{J_0^2\left(x_\nu \dfrac{R_2}{R_w}\right) - J_0^2(x_\nu)}\, U_\nu\left(x_\nu \frac{r}{R_w}\right) \exp\left(-x_\nu^2 \frac{\chi_2 t}{R_w^2}\right), \tag{7}$$

where

$$U_\nu\left(x_\nu \frac{r}{R_w}\right) = J_0\left(x_\nu \frac{r}{R_w}\right) Y_0\left(x_\nu \frac{R_1}{R_w}\right) - J_0\left(x_\nu \frac{R_1}{R_w}\right) Y_0\left(x_\nu \frac{r}{R_w}\right),$$

and meets condition (2), where $\Delta P_{st} = P_s - P_f$.

The rate of liquid flow is determined by the following equality:

$$Q = -2\pi r h \frac{k_2}{\mu} \frac{\partial \Delta P_2(r, t)}{\partial r}\bigg|_{r=R_w}\,, \tag{8}$$

solving which, with allowance for (7), we obtain

$$Q_w = -\frac{2\pi R_w h k_2 \Delta P_{st}}{\mu \ln \dfrac{R_w}{R_2}} \left[\frac{1}{R_w} - \frac{\pi}{R_w}\left(\ln \frac{R_2}{R_w}\right) \sum_{\nu=1}^{\infty} \frac{x_\nu J_0\left(x_\nu \dfrac{R_2}{R_w}\right) J_0\left(x_\nu \dfrac{R_1}{R_w}\right)}{J_0^2\left(x_\nu \dfrac{R_2}{R_w}\right) - J_0^2(x_\nu)} \times\right.$$

$$\left. \times \left(J_1(x_\nu)\, Y_0(x_\nu) - J_0(x_\nu)\, Y_1(x_\nu)\right) \exp\left(-x_\nu^2 \frac{\chi_2 t}{R_w^2}\right)\right]. \tag{9}$$

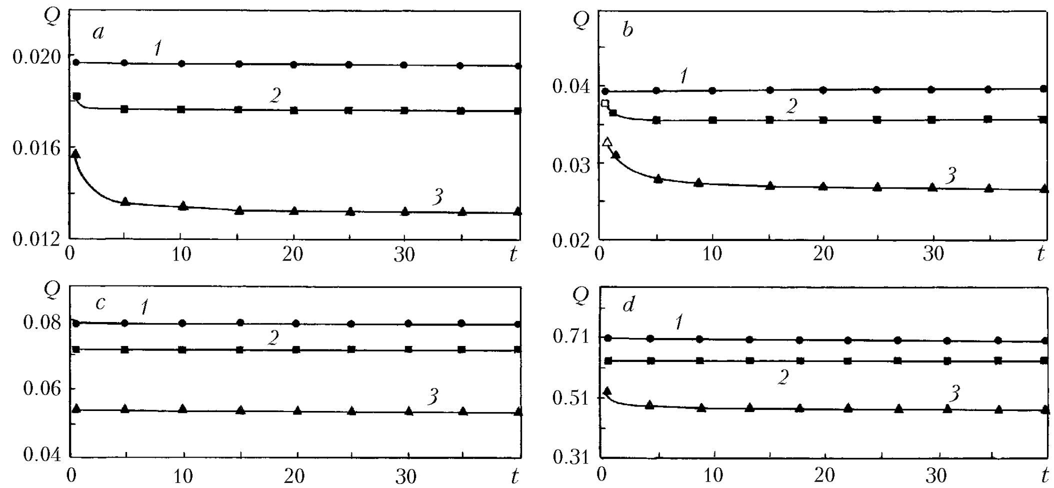

Fig. 2. Change in the rate of liquid flow in inhomogeneous porous media with time: a, c) filtration in a medium with decreasing permeability; b, d) filtration in a medium with increasing permeability; a) $k_1 = 0.1 \cdot 10^{-12}$ m^2, $k_2 = 0.05 \cdot 10^{-12}$ m^2; b) $k_1 = 0.05 \cdot 10^{-12}$ m^2, $k_2 = 0.1 \cdot 10^{-12}$ m^2; c) $k_1 = 1.8 \cdot 10^{-12}$ m^2, $k_2 = 0.2 \cdot 10^{-12}$ m^2; d) $k_1 = 0.2 \cdot 10^{-12}$ m^2, $k_2 = 1.8 \cdot 10^{-12}$ m^2 [R_2/R_1 = 5 (1), 10 (2), and 100 (3)]. Q, 10^{-3} m^3/sec; t, sec.

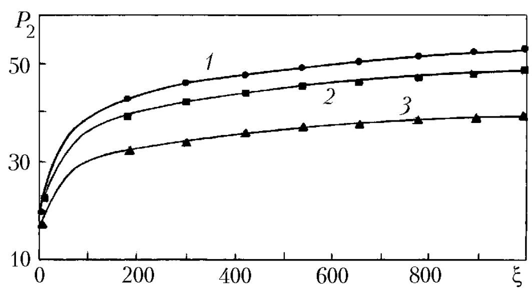

Fig. 3. Dependence of the pressure P_2 on the coordinate ξ: R_2/R_1 = 5 (1), 10 (2), and 100 (3). P_2, MPa.

TABLE 1. Data for Drowned Wells of the "Oil Stones" Deposit Operating in the Pumping Regime

Conditional numbers of wells	$Q_p \cdot 10^5$, m^3/sec	$Q_{pr} \cdot 10^5$, m^3/sec	$K_r = (Q_p/\Delta P) \cdot 10^4$, m^3/(sec·MPa)	$K_{prod} = (Q_{pr}/\Delta P) \cdot 10^4$, m^3/(sec·MPa)	K_r/K_{prod}
1	98.95	1.11	2.41	0.55	4.38
2	110.18	2.68	2.68	1.34	2.00
3	141.43	3.15	3.36	1.57	2.14
4	161.92	0.47	3.68	0.24	15.33
5	155.67	1.75	3.54	0.87	4.06

Figures 2 and 3 present the results of numerical calculations performed by formulas (7) and (9) at the following values of the parameters: P_s = 50 MPa, P_f = 10 MPa, ΔP_{st} = 40 MPa, R_w = 0.085 m, R_1 = [10, 100] m, R_2 = [50, 5000] m, χ_1 = 0.3 and 6 m^2/sec, χ_2 = 0.17 and 0.66 m^2/sec, $k_1 = 0.1 \cdot 10^{-12}$ and $1.8 \cdot 10^{-12}$ m^2, $k_2 = 0.05 \cdot 10^{-12}$ and $0.2 \cdot 10^{-12}$ m^2, μ = 1 cP, h = 10 m, and ξ = [1, 1000].

As is seen from Fig. 2a and b, with decrease in the permeability of a medium in which a liquid flow is filtrated, the rate of this flow decreases by almost two times and, at $k_1 = 1.8 \cdot 10^{-12}$ m^2 and $k_2 = 0.2 \cdot 10^{-12}$ m^2 (Fig. 2c and d), by almost ten times as compared to that of the reverse flow, which correlates with the experimental data (see Table 1).

It follows from Fig. 3 that the pressure field in the second zone depends substantially on the ratio R_2/R_1, and the loss in the pressure increases with increase in this ratio.

NOTATION

A_ν, B_ν, D_1, D_2, integration constants; h, thickness of the seam, m; J, modified Bessel function of the zero order; K_{prod}, coefficient of productivity, $m^3/(sec \cdot MPa)$; K_r, coefficient of responsiveness ($m^3/sec \cdot MPa$); k_1 and k_2, permeability of the porous medium in the first and second zones of the seam, m^2; P_1 and P_2, pressure in the first and second zones of the seam, MPa; ΔP_1 and ΔP_2, pressure drop in the first and second zones of the seam, MPa; P_f, pressure in the face of the well, MPa; P_s, pressure in the supply loop, MPa; ΔP_{st}, differential pressure between the supply loop and the face of the well, MPa; Q, rate of liquid flow, m^3/sec; Q_{pr}, rate of produced liquid flow, m^3/sec; Q_p, rate of pumped-liquid flow, m^3/sec; Q_w, rate of liquid flow through the wall of the well, m^3/sec; r, coordinate; R_1 and R_2; radius of the first and second zones of the seam, m; R_w, radius of the well, m; t, time; sec; U_ν, function, $\nu = 1, 2, 3, \ldots$; x_ν, root of the transcendental equation; Y_0, modified Bessel function of the zero order; χ_1 and χ_2, coefficients of piezoconduction in the first and second zones of the seam, m^2/sec; μ, dynamic viscosity of liquid, cP; $\xi = r/R_w$, coordinate. Subscripts: pr, produced; f, face; p, pumped; s, supply; prod, productivity; r, responsiveness; w, well; st, stationary; 0, zero order; 1 and 2, first and second zones of the seam.

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